Linear Feedback Control Using Quasi Velocities

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A novel approach for designing feedback controllers for natural mechanical systems using quasi velocities is presented. The approach is relevant to the stabilization and regulation of finite-dimensional multibody systems. In particular the globally asymptotically stable linear feedback of Rodrigues parameters and angular velocity is taken from spacecraft attitude control and applied to a broader class of problems. This controller is shown to have good performance due to the sensitivity of the rotational kinematics to small motions near the reference configuration. Additionally, the concept is extended to systems with more than three degrees of freedom by generalizing the functional form of the three-dimensional rotational kinematics. This defines a new set of quasi velocities that allow globally asymptotically stable linear feedback for any number of degrees of freedom. An example shows that the use of these variables in controller design can lead to improved performance.

I. Introduction

BEGINNING with Euler, the rotational motion of three-dimensional bodies has been studied for over 200 years. The development of spacecraft technology over the past 50 years has inspired a continuing focus on this problem that has produced many significant developments in attitude representations and control. Most of these control approaches have used the angular velocity, which is a set of quasi velocities [1,2] for rotational motion. An example of this, that will be a focus of this paper, is the proof discovered by Tsiotras for global asymptotic stability using linear feedback of angular velocity and the Rodrigues parameters [3,4].

The use of quasi velocities in designing feedback controllers can also be applied to other mechanical systems. Careful selection of the motion variables used in controller design is an example of the interaction between dynamics and control [5]. Previously, Schaub and Junkins investigated feedback control in terms of a set of eigenfactor quasi velocities [6]. These quasi velocities are defined by an orthogonal decomposition of the system mass matrix. An advantage of these variables is that they produce controllers that decouple the individual modes of the system dynamics.

Another issue related to the choice of variables and controller design concerns the globality of global asymptotic stability. Conventional stability definitions are stated in terms of a particular set of coordinates. The "global stability" of a coordinate set, however, is a separate issue from whether or not those coordinates "globally" describe the configuration of the underlying physical system. In fact, the Rodrigues parameters are an illustrative example, because they do not globally describe the rotation group. In a particular application, global stability of the Rodrigues parameters may or may not result in the desired physical behavior.

In this paper, rotational kinematics will be used to define a set of quasi velocities that allows direct application of attitude controllers to a broad class of 3-degree-of-freedom (DOF) problems. To extend this concept to higher DOF problems, however, new quasi velocities must be developed. These quasi velocities and this approach can be used for any time-independent, finite-dimensional, multibody system undergoing general motion.

Section II of this paper describes the particular control problem that will be considered. Section III reviews some elegant attitude control results discovered by Tsiotras, approaching them as a special case of the considered problem. The controller developed by Tsiotras is applied to a three-link manipulator system in Sec. IV. Moving to higher DOF systems, Sec. V presents an alternative set of quasi velocities that can be used in control of higher DOF systems. These quasi velocities are applied to the control of a four-link manipulator system in Sec. VI.

II. Problem Definition

Consider a natural, mechanical system described by some set of generalized coordinates q, with kinetic energy $T(q, \dot{q}) = \frac{1}{2}\dot{q}^\mathsf{T} M(q)\dot{q}$. This type of system is called natural because the kinetic energy contains only terms that are quadratic in the generalized velocities. The systems considered here are also time-independent. The choice to focus on this type of system flows from the choice to focus on feedback control. The methods developed in this paper could probably be adapted to time-dependent systems (e.g., systems with prescribed motions); however, this would likely require the addition of control terms related to the time dependency.

It will be assumed that all external torques and forces applied to the system, both potential and nonpotential, are described by the generalized forces f associated with the generalized coordinates. (Note that this assumption can be somewhat relaxed by the use of feedback linearization and other techniques.) Additionally, the generalized forces will be treated as control variables. Significantly, this assumes a fully actuated system wherein there is one control actuator for each system degree of freedom.

This paper will consider the design of linear globally asymptotically stable controllers for natural systems subject to generalized forces using the following Lyapunov function.

$$V(q, \dot{q}) = T(q, \dot{q}) + k\phi(q) \qquad (1)$$

Here, k is a positive control gain. For V to be a valid Lyapunov function, ϕ must be a positive-definite function of the generalized coordinates. For the given system, the time rate of change of the kinetic energy is the power of the generalized forces. Therefore, the derivative of V is given by the following.

$$\dot{V} = \dot{q}^\mathsf{T} f + k\left(\frac{\partial\phi}{\partial q}\right)^\mathsf{T}\dot{q} \qquad (2)$$

Clearly, a linear globally asymptotically stable controller results from choosing $\phi = \frac{1}{2}q^\mathsf{T} q$ and the following control law.

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$$f = -k(\dot{q} + q) \qquad (3)$$

Hence, this controller globally stabilizes and regulates the system driving it toward $q = \dot{q} = \mathbf{0}$.

Whereas the preceding problem in terms of generalized velocities is trivial, a more interesting problem, and the focus of this paper, deals with quasi velocities. The aim of this approach will be to find globally asymptotically stable control laws for the generalized forces in terms of the quasi velocities, called the quasi forces. Additionally, the desired controllers will be linear feedback of the quasi velocities and generalized coordinates. A controller of this form is chosen mainly for purposes of comparison with the linear feedback of generalized coordinates and velocities. Other controllers could also be investigated; however, the chosen form does offer some elegance in globally controlling a nonlinear system with a linear control law. This form can be contrasted with feedback linearization, which attempts to find a nonlinear controller that produces a linear closed-loop system.

In this paper, a vector of quasi velocities $\boldsymbol{\omega}$ will be a linear transformation of the generalized velocities as shown in Eq. (4).

$$\dot{q} = A(q)\omega; \qquad \omega = B(q)\dot{q} \qquad (4)$$

Here, $B = A^{-1}$. The quasi forces are defined as a similar linear transformation of the generalized forces.

$$f^{(\omega)} = A^{\mathsf{T}} f; \qquad f = B^{\mathsf{T}} f^{(\omega)} \qquad (5)$$

Equations (4) and (5) can be used to write the work/energy-rate expression for any set of quasi velocities, once and for all.

$$\dot{T} = \dot{q}^{\mathsf{T}} f = \omega^{\mathsf{T}} A^{\mathsf{T}} B^{\mathsf{T}} f^{(\omega)} = \omega^{\mathsf{T}} f^{(\omega)} \qquad (6)$$

The derivative of the kinetic energy is the power of any set of quasi forces.

Equation (2) can now be rewritten in terms of quasi velocities and quasi forces.

$$\dot{V} = \omega^{\mathsf{T}} f^{(\omega)} + k \left(\frac{\partial \phi}{\partial q}\right)^{\mathsf{T}} A\omega \qquad (7)$$

A globally asymptotically stable controller can clearly be defined by choosing the following quasi forces.

$$f^{(\omega)} = -k\left(\omega + A^{\mathsf{T}} \frac{\partial \phi}{\partial q}\right) \qquad (8)$$

Compared with the generalized-velocity control, however, finding a linear expression for the quasi forces is somewhat more difficult. For linear feedback, a function ϕ must be found for any particular set of quasi velocities such that $\partial \phi / \partial q = B^{\mathsf{T}} q$. An example of such a function, which was discovered by Tsiotras, is discussed in the next section.

III. Linear Angular Velocity Feedback

Perhaps the best known example of quasi velocities is the angular velocity of rotational motion. These velocities are extremely useful in both the dynamics and control of rigid-body rotations. In this section their relation to the Rodrigues parameters, a common choice for generalized coordinates of rotational motion, is examined. The kinematic transformation matrix A for these generalized coordinates and quasi velocities is shown in Eq. (9) .

$$A = \frac{1}{2}(I + Q + qq^{\mathsf{T}}) \qquad (9)$$

Here, Q is the 3×3, skew-symmetric matrix representation of the elements of q. In this section, several special properties of this three-dimensional form will be demonstrated that lead to a proof of global asymptotic stability for linear feedback of the Rodrigues parameters and angular velocity. First, it will be shown that q is an eigenvector of A^{T}, using the fact that the product Qq equals zero.

$$A^{\mathsf{T}} q = \frac{1}{2}(q - Qq + qq^{\mathsf{T}} q) = \frac{1}{2}(1 + q^{\mathsf{T}} q)q \equiv \lambda q \qquad (10)$$

Therefore, the eigenvalue associated with q is $\frac{1}{2}(1 + q^{\mathsf{T}} q)$.

A similar derivation can be used to show that q is also an eigenvector of A. This fact, however, can also be understood from a physical interpretation. Consider the situation of ω being aligned with q, $\omega = \alpha q$. In this case the body is simply "spinning up" because the vector of Rodrigues parameters is always parallel with the principal axis of rotation. Therefore, the direction of q is constant, and only its magnitude is changing. This means that $\dot{q}$ is also parallel to q, $\dot{q} = \alpha \lambda q$. In this case, the first of Eqs. (4) becomes the following.

$$\alpha \lambda q = A(\alpha q) \qquad (11)$$

The proportionality factors α and λ are simply scalars, and thus q being an eigenvector of A is physically expected.

Equation (10) allows the following elegant proof discovered by Tsiotras [3]. Consider the following function ϕ.

$$\phi = \ell_n(1 + q^{\mathsf{T}} q) \qquad (12)$$

This leads to the following result for the second term of the Lyapunov function derivative.

$$A^{\mathsf{T}} \frac{\partial \phi}{\partial q} = 2\frac{A^{\mathsf{T}} q}{1 + q^{\mathsf{T}} q} = q \qquad (13)$$

The following linear controller is therefore globally asymptotically stable.

$$f^{(\omega)} = -k(\omega + q) \qquad (14)$$

For rigid-body rotations, the quasi forces associated with the angular velocity are the moments applied to the body.

IV. Three-Link Manipulator Example

The controller in the preceding section was developed by Tsiotras in terms of Rodrigues parameters and angular velocities with the spacecraft attitude-control application in mind. The same controller, however, can be applied to a broad variety of systems in terms of generalized coordinates and quasi velocities. In the preceding section, the kinematics in Eq. (9) defined the angular velocities of rotational motion. In this section, the same kinematics are applied to general motions defining a new set of quasi velocities, called the Cayley quasi velocities [9]. Furthermore, in this section the control law in Eq. (14) is applied to general motions and is referred to as the Cayley quasi-velocity control. This approach will be illustrated through application to a three-link manipulator system.

Whereas the quasi-velocity control was originally given in terms of the quasi forces, this can be easily converted to an expression for the generalized forces.

$$f = B^{\mathsf{T}} f^{(\omega)} = -k_2(B^{\mathsf{T}} \omega + B^{\mathsf{T}} q) = -k_2(B^{\mathsf{T}} B\dot{q} + B^{\mathsf{T}} q) \qquad (15)$$

Here, the control gain has been relabeled k_2. Equation (15) is referred to as the $\boldsymbol{\omega}$ control law. The performance of this controller will be compared with the linear feedback controller designed directly using generalized velocities, with control gain k_1.

$$f = -k_1(\dot{q} + q) \qquad (16)$$

This is referred to as the $\dot{q}$ control law.

Following Schaub and Junkins [6], the controllers in Eqs. (15) and (16) are applied to a serial three-link manipulator system in the plane with no gravity acting. The generalized coordinates of the system are the absolute angles of each joint.

$$[q] = \begin{bmatrix} \theta_1 & \theta_2 & \theta_3 \end{bmatrix}^{\mathsf{T}}$$

The generalized forces are related to the motor torques acting at each joint, with no other externally applied forces or torques acting. The

system mass matrix was given by Schaub and Junkins.

$$[\boldsymbol{M}] = \begin{bmatrix} (m_1 + m_2 + m_3)l_1^2 & (m_2 + m_3)l_1 l_2 \cos(\theta_2 - \theta_1) & m_3 l_1 l_3 \cos(\theta_3 - \theta_1) \\ (m_2 + m_3)l_1 l_2 \cos(\theta_2 - \theta_1) & (m_2 + m_3)l_2^2 & m_3 l_2 l_3 \cos(\theta_3 - \theta_2) \\ m_3 l_1 l_3 \cos(\theta_3 - \theta_1) & m_3 l_2 l_3 \cos(\theta_3 - \theta_2) & m_3 l_3^2 \end{bmatrix} \qquad (17)$$

Here, m_1, m_2, and m_3 are the masses of point masses located at the tip of each link, and l_1, l_2, and l_3 are the lengths of each link.

Simulations were performed, selecting initial coordinates and velocities.

$$[\boldsymbol{q}(0)] = \begin{bmatrix} 10 & 10 & 10 \end{bmatrix}^{\mathsf{T}} \text{ deg}$$

$$[\dot{\boldsymbol{q}}(0)] = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}^{\mathsf{T}} \text{ deg}/s$$

The values m_1, m_2, m_3, l_1, l_2, and l_3 were set to one. The control gains were chosen for each controller such that the maximum control efforts encountered were equal. The values $k_1 = 1$ and $k_2 = 0.54$ were chosen. The numerical integration was performed using a fourth and fifth order Runge–Kutta method and a simulation duration of 50 time units.

The results from the simulation are shown in Figs. 1–3. The state angles and kinetic energy show the performance of each controller in
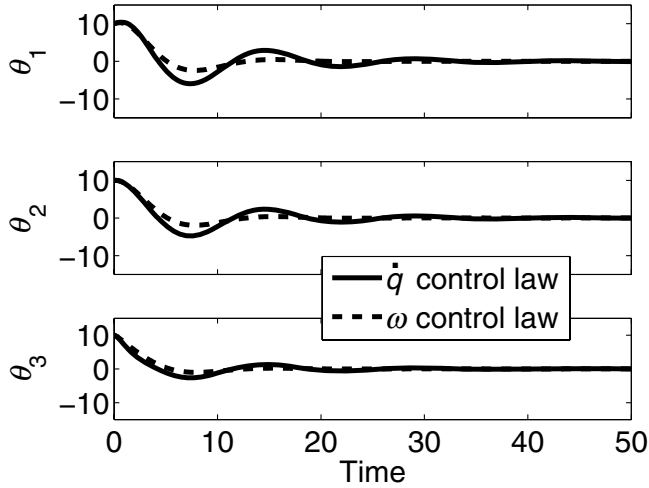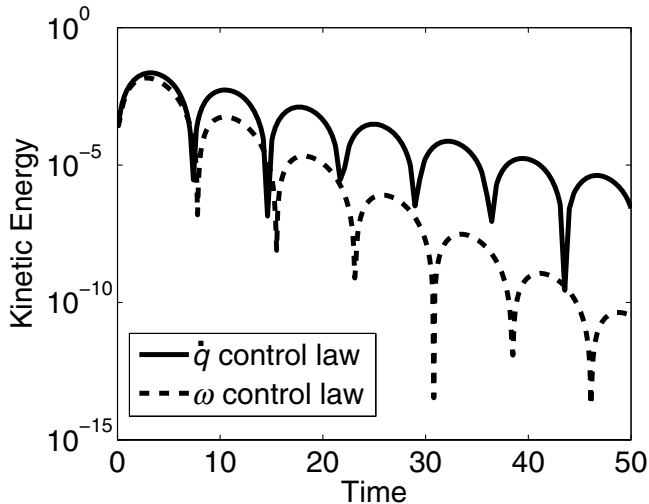
driving the system to rest at the desired configuration. These results demonstrate the stability of both controllers; however, the $\boldsymbol{\omega}$ control law clearly has faster convergence. A slightly lower frequency in the $\boldsymbol{\omega}$ control response is also noticeable. Figure 3 shows that both controllers used the same maximum control effort, $\|\boldsymbol{f}\|_2$, at the initial time. After the initial time, the necessary control effort for the $\boldsymbol{\omega}$ control law is lower due to the quicker convergence.

In addition to investigating simulation time histories, the two controllers can also be compared by performing a linearized analysis. Linearization about $\boldsymbol{q} = \dot{\boldsymbol{q}} = \boldsymbol{0}$ produces the following linearized closed-loop systems.

$$\boldsymbol{M}_0 \ddot{\boldsymbol{q}} + k_1 \dot{\boldsymbol{q}} + k_1 \boldsymbol{q} = \boldsymbol{0} \qquad (18)$$

$$\boldsymbol{M}_0 \ddot{\boldsymbol{q}} + 4k_2 \dot{\boldsymbol{q}} + 2k_2 \boldsymbol{q} = \boldsymbol{0} \qquad (19)$$

Here, Eq. (18) is associated with the $\dot{\boldsymbol{q}}$ control law, Eq. (19) is associated with the $\boldsymbol{\omega}$ control law, and $\boldsymbol{M}_0$ is the evaluation of the mass matrix at $\boldsymbol{q} = \boldsymbol{0}$. For the gains and system values used, the dominant poles of the system in Eq. (18) are $-0.0990 \pm 0.4339i$. For Eq. (19), the dominant poles are $-0.2139 \pm 0.4101i$. These poles reflect a faster convergence rate for the $\boldsymbol{\omega}$ control law.

The lower kinetic energy and quicker convergence with the $\boldsymbol{\omega}$ control law indicates that, for this particular example, stabilization using the Cayley quasi velocities outperforms the generalized velocities. The $\boldsymbol{\omega}$ control law adds a state-dependent influence matrix to the velocity feedback. This influence matrix is related to the kinematic transformation $\boldsymbol{B}$ and produces greater control sensitivity and damping at the point of interest, $\boldsymbol{q} = \boldsymbol{0}$.

The performance demonstrated in this section for the controller designed using the Cayley quasi velocities motivates the idea of applying this concept to other systems. This approach, of course, used rotational kinematics to define a set of quasi velocities used in feedback control. Whereas this approach is clear for 3-DOF systems, the concept can also be extended to more complex systems with greater degrees of freedom. In the following section an approach is presented based on defining a new set of quasi velocities.
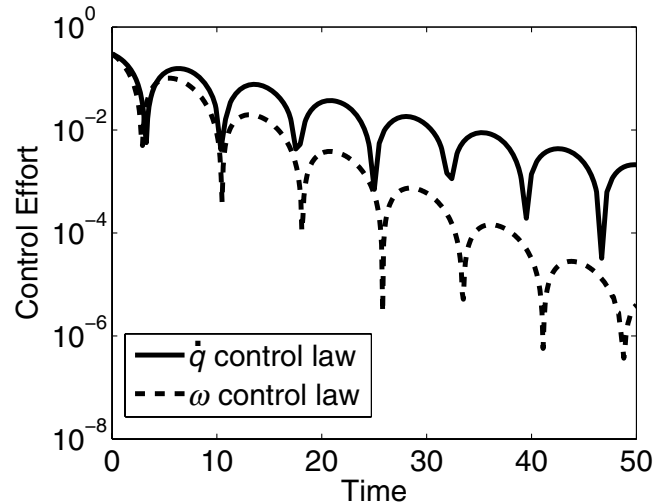

Fig. 1 Configuration angles time histories in degrees.

Fig. 2 Kinetic energy time history.

Fig. 3 Control vector magnitude time history.

V.   Quasi Velocities for Linear Feedback

In Sec. III several special properties of the angular-velocity/ Rodrigues-parameter kinematics were discussed that led to a proof for global asymptotic stability of linear feedback. Those kinematics are described by the transformation matrix A, repeated here for convenience.

$$A = \frac{1}{2}(I + Q + qq^\mathsf{T}) \qquad (20)$$

This section presents an alternative set of quasi velocities that allows the proof to be extended to systems with an arbitrary number of DOFs (i.e., M-DOF systems). The new quasi velocities are defined by simply modifying the functional form of Eq. (20).

To do this a new $M \times M$ transformation matrix $\bar{A}$ will be defined. Two difficulties exist, however, in directly extending the functional form of Eq. (20) to any M-DOF system. Both are directly related to the second term, Q. An $N \times N$ skew-symmetric matrix has only M^* distinct elements, where $M^* = N(N-1)/2$. Therefore, Q can only be constructed for systems with $M = M^* \in \{1, 3, 6, 10, \ldots\}$. Moreover, Q cannot, in general, be added to qq^T, for this is only possible when $M = N = 3$. This suggests the following mapping to define a new set of quasi velocities u.

$$\dot{q} = \bar{A}u; \qquad u = \bar{B}\dot{q} \qquad (21)$$

$$\bar{A} = \frac{1}{2}(I + qq^\mathsf{T}) \qquad (22)$$

This set of quasi velocities can be applied to a general system with any number of generalized coordinates. For Eqs. (21) and (22) to constitute a valid quasi-velocity definition, however, the inverse of $\bar{A}$ (i.e., $\bar{B}$) must exist. The matrix $2\bar{A}$ is equal to $I + qq^\mathsf{T}$, and the

$$f^{(u)} = -k(u + q) \qquad (26)$$

Applying this control law to the three-link manipulator example in Sec. IV produces results very similar to the results presented for the Cayley quasi velocities. The advantage of the new quasi velocities, however, is that they can be applied to more complex systems with any number of degrees of freedom. In the following section, the new control law will be applied to a four-link manipulator system.

VI.   Four-Link Manipulator Example

To apply the new quasi-velocity controller to a mechanical system, the control law is first converted to an expression in generalized forces, relabeling the control gain as k_3.

$$f = \bar{B}^\mathsf{T} f^{(u)} = -k_3(\bar{B}^\mathsf{T} u + \bar{B}^\mathsf{T} q) = -k_3(\bar{B}^\mathsf{T}\bar{B}\dot{q} + \bar{B}^\mathsf{T} q) \qquad (27)$$

This control law is called the u control law and will be compared with the generalized-velocity control in Eq. (16).

Because $\bar{B}$ can be defined for any number of degrees of freedom, the controllers will now be applied to a four-link manipulator. This mechanism is similar to the three-link example with an additional joint, link, and tip mass added to the end. Similar to the three-link example, the generalized coordinates of the four-link system are absolute angles of the joints.

$$[q] = \begin{bmatrix} \theta_1 & \theta_2 & \theta_3 & \theta_4 \end{bmatrix}^\mathsf{T}$$

The new system mass matrix is shown in Eq. (28).

$$[M] = \begin{bmatrix} (m_1 + m_2 + m_3 + m_4)l_1^2 & (m_2 + m_3 + m_4)l_1l_2\cos(\theta_2 - \theta_1) & (m_3 + m_4)l_1l_3\cos(\theta_3 - \theta_1) & m_4l_1l_4\cos(\theta_4 - \theta_1) \\ (m_2 + m_3 + m_4)l_1l_2\cos(\theta_2 - \theta_1) & (m_2 + m_3 + m_4)l_2^2 & (m_3 + m_4)l_2l_3\cos(\theta_3 - \theta_2) & m_4l_2l_4\cos(\theta_4 - \theta_2) \\ (m_3 + m_4)l_1l_3\cos(\theta_3 - \theta_1) & (m_3 + m_4)l_2l_3\cos(\theta_3 - \theta_2) & (m_3 + m_4)l_3^2 & m_4l_3l_4\cos(\theta_4 - \theta_3) \\ m_4l_1l_4\cos(\theta_4 - \theta_1) & m_4l_2l_4\cos(\theta_4 - \theta_2) & m_4l_3l_4\cos(\theta_4 - \theta_3) & m_4l_4^2 \end{bmatrix} \qquad (28)$$

eigenvalues of $2\bar{A}$ are all greater than or equal to positive one. Therefore, $\bar{A}$ is nonsingular, and u is a valid set of quasi velocities.

To design a controller in terms of these new quasi velocities, Eq. (8) is rewritten in terms of u and $\bar{A}$.

$$f^{(u)} = -k\left(u + \bar{A}^\mathsf{T}\frac{\partial\phi}{\partial q}\right) \qquad (23)$$

Linear feedback of q and u can be proven to be globally asymptotically stable, using the same Lyapunov function defined by Tsiotras. Because of the choice of $\bar{A}$, however, q is now an eigenvector of $\bar{A}^\mathsf{T}$ for any value of M.

$$\bar{A}^\mathsf{T} q = \frac{1}{2}(q + qq^\mathsf{T}q) = \frac{1}{2}(1 + q^\mathsf{T}q)q \equiv \lambda q \qquad (24)$$

This leads to the following result for the second term of the control law.

$$\bar{A}^\mathsf{T}\frac{\partial\phi}{\partial q} = 2\frac{\bar{A}^\mathsf{T} q}{1 + q^\mathsf{T}q} = q \qquad (25)$$

The following linear controller for the quasi forces associated with u is therefore globally asymptotically stable.

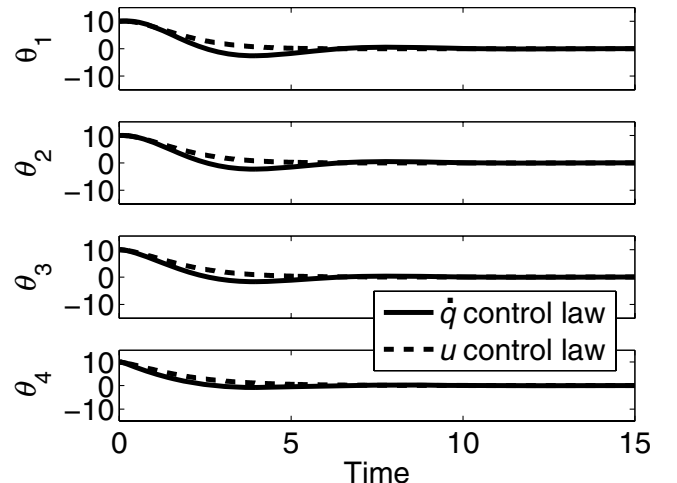Simulations were performed, selecting initial coordinates and velocities.

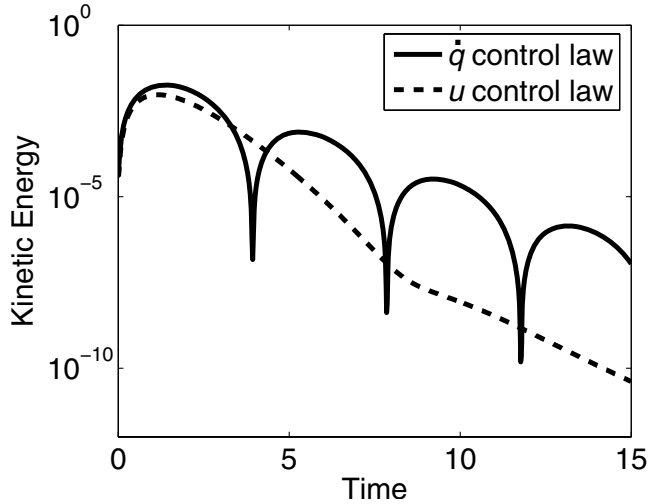Fig. 4   Configuration angles time histories in degrees.
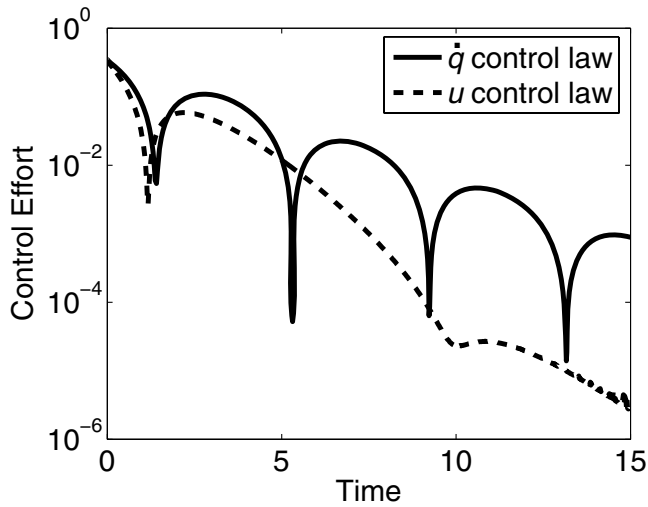
Fig. 5   Kinetic energy time history.

Fig. 6   Control vector magnitude time history.

$$[\boldsymbol{q}(0)] = \begin{bmatrix} 10 & 10 & 10 & 10 \end{bmatrix}^{\mathrm{T}} \text{ deg},$$

$$[\dot{\boldsymbol{q}}(0)] = \begin{bmatrix} 1 & 0 & -1 & 0 \end{bmatrix}^{\mathrm{T}} \text{ deg}/\text{s}$$

Similar to the preceding simulation, the lengths of each link were again set to one; however, to explore different parameter values the masses were set to 0.15. The control gains were again chosen for each controller such that the maximum control efforts encountered were equal. Because of the similarity of the $\boldsymbol{u}$ and $\boldsymbol{\omega}$ control laws, the values $k_1 = 1$ and $k_3 = 0.54$ were again used. The system equations were integrated for a simulation duration of 15 time units.

The results from the simulation are shown in Figs. 4–6. The state angles and kinetic energy show faster convergence of the $\boldsymbol{u}$ control law. More dramatically, however, the performance for the $\boldsymbol{u}$ control law has much greater damping than the $\boldsymbol{q}$ control law. The behavior of the $\boldsymbol{u}$ control law appears similar to a critically damped linear system.

Similar to the Cayley quasi-velocity control, the superior performance of the $\boldsymbol{u}$ control law is related to the kinematic transformation $\bar{\boldsymbol{B}}$. The matrices $\boldsymbol{B}(\boldsymbol{q})$ and $\bar{\boldsymbol{B}}(\boldsymbol{q})$ are similar in that both approach $\boldsymbol{0}$ for large values of $\boldsymbol{q}$, and both approach $2\boldsymbol{I}$ for values of $\boldsymbol{q}$ near the origin $\boldsymbol{q} = \boldsymbol{0}$. Considering the kinematic equations, this growth in $\boldsymbol{B}$ and $\bar{\boldsymbol{B}}$ near the origin indicates that small values of $\dot{\boldsymbol{q}}$ produce large values of $\boldsymbol{\omega}$ and $\boldsymbol{u}$. Therefore, the quasi velocities are sensitive to small motions near the origin.

Examining Eqs. (15) and (27) demonstrates the impact of this kinematic scaling on the respective control laws. The scaling applied

to $\boldsymbol{q}$ and $\dot{\boldsymbol{q}}$ in these laws produces a "gain-scheduling" effect that provides greater sensitivity near $\boldsymbol{q} = \boldsymbol{0}$. This allows for quicker convergence near the origin without requiring larger control effort far from the origin. Additionally, because the $\dot{\boldsymbol{q}}$ term is scaled double the amount of the generalized coordinate term, damping is increased near the origin. This allows for well-damped behavior near the origin without slowing the system down far from the origin.

Of course, typical gain-scheduled controllers are developed by explicitly interpolating the gains chosen for a family of linearized design points [10]. The performance shown here for the quasi-velocity control is achieved without any explicit manipulation of the gains, and is instead a result of the implicit properties of the quasi velocities. Additionally, the controllers designed here apply globally and do not rely on any linearization assumptions.

VII.   Conclusion

A novel approach for the design of feedback controllers for natural mechanical systems has been demonstrated in this paper. This was done by taking concepts from spacecraft attitude control and generalizing them to apply to the broad class of fully actuated natural mechanical systems. This approach developed control expressions for quasi forces that were linear in the quasi velocities and generalized coordinates. These quasi-velocity controllers were shown for two example problems to have performance superior to linear feedback of the generalized velocities. The performance benefits of these controllers were not achieved by explicitly modifying the control law, but instead are related to the implicit properties of the motion variables used in the control design.

As mentioned, this performance is related to the kinematics that define the quasi velocities. For the Cayley quasi velocities these kinematics are given by the kinematics of a rigid body and reflect the property that, for small values of the Rodrigues parameters, small rates are equivalent to large angular velocities. Although the alternative quasi velocities do not have such a geometric interpretation, they share the same sensitivity to small motions near the origin. It is this sensitivity that results in the superior performance of the controllers designed in terms of these variables.

Although only a few specific cases were investigated in this paper, there is reason to believe that similar performance advantages might also be seen in the regulation of other multibody systems. The properties of the quasi velocities presented here could make them particularly suited for general regulation problems. Furthermore, the robustness of controllers designed using the new quasi velocities has not been investigated; however, these variables do have some attractive features in this regard. Unlike some other quasi velocities, the definitions presented here are not dependent on knowledge of system parameters. Additionally, the gain-scheduling effect of the variables allows good performance with lower overall gain settings. Both of these features suggest that the approach presented here might lead to good robustness characteristics.

Taken from a broader view, the results in this paper are further illustration of the importance of the interaction between dynamics and control. Instead of searching for new control laws for a particular set of motion variables, improved controllers can also be designed by substituting new variables into a given expression. In this case, it is intuitively desirable for controller design to use motion variables that are sensitive to motions about the point of interest.

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